UV photon splits either b or c and causes downconversion in either $X_1^{(2)}$ or $X_2^{(2)}$

Let's consider upper route

Let's consider lower route

These two "parts" interfere
\[ I_s = \cos^2 \varphi \]

We see interference between upper and lower path at \( D_s \) even if \( D_l \) is not there.

\( D_l \) cannot tell if photon came from \( x^{(1)} \) or \( x^{(2)} \).

Now we block beam \( E \) with a beam block. \( D_l \) can now tell which path (lower) and interference at \( D_s \) disappears.

Note: Block 'B' is not in either path.

Note: \( D_l \) does not actually need to be there! Only potential.
\[ |\psi_\pi\rangle = \frac{1}{\sqrt{2}} \left[ |1\rangle_\nu |0\rangle_c + e^{i\phi} |0\rangle_\nu |1\rangle_c \right] \]

\[ |\psi_\Pi\rangle = \frac{1}{\sqrt{2}} \left[ |1\rangle_\nu |0\rangle_c + e^{i\phi} |1\rangle_\nu |1\rangle_c \right] \]

\[ \hat{D}_{C_1} |1\rangle_\nu |0\rangle_c |0\rangle_e = \gamma |0\rangle_\nu |0\rangle_c |1\rangle_d |1\rangle_e \]

\[ \hat{D}_{C_2} |0\rangle_\nu |1\rangle_c |0\rangle_h |0\rangle_k = \gamma |0\rangle_\nu |0\rangle_c |1\rangle_h |1\rangle_k \]

Note: If mode $e$ and $k$ are aligned, they are the same mode!

\[ \hat{D}_{C_2} \hat{D}_{C_1} |\psi_\Pi\rangle = |\psi_\Pi\rangle \]

\[ \hat{\Sigma}_\phi |1\rangle_h = |1\rangle_h \quad \text{and} \quad \hat{\Sigma}_\phi |1\rangle_h = e^{i\phi} |1\rangle_h \]

\[ |\psi_\Pi\rangle = \left[ |1\rangle_\nu |1\rangle_\phi |0\rangle_c + e^{i\phi} |0\rangle_\nu |0\rangle_c |1\rangle_\phi |1\rangle_c \right] \]
Now: \[ \hat{B}_c \left| 11 \right>_m \left| 0 \right>_e \] = \frac{1}{\sqrt{2}} \left[ \left| 11 \right>_m \left| 0 \right>_e + i \left| 10 \right>_m \left| 11 \right>_e \right] \]

\[ \hat{B}_c \left| 10 \right>_m \left| 11 \right>_e \] = \frac{1}{\sqrt{2}} \left[ \left| 10 \right>_m \left| 11 \right>_e + i \left| 11 \right>_m \left| 10 \right>_e \right] \]

\[ \psi_{\psi} = \hat{B}_c \psi \]
\[ = \frac{1}{\sqrt{2}} \left[ \left| 11 \right>_m \left| 0 \right>_e \right] \left( \left| 11 \right>_m \left| 0 \right>_e + i \left| 10 \right>_m \left| 11 \right>_e \right) \]
\[ + i e^{i \phi} \left| 10 \right>_m \left| 11 \right>_e \right] \left( \left| 10 \right>_m \left| 11 \right>_e + i \left| 11 \right>_m \left| 10 \right>_e \right) \]

And no beam block \( B \)

Now if aligned \( 1 \left| e \right. \) and \( 1 \left| k \right. \)

the same mode and \( \left| 11 \right>_m \left| 0 \right>_e \left| k \right. = \left| 10 \right>_m \left| 11 \right>_e \equiv \left| 11 \right>_e \)

That is you can not tell even in principle is photon \( 1 \left| k \right. \) was created in \( B_c \, C_{1012} \)

Hence it factorizes!

\[ \psi = \frac{e^{i \phi}}{\sqrt{2}} \left| 11 \right>_e \left[ \left( \frac{e^{-i \phi} - e^{i \phi}}{2} \right) \left| 11 \right>_m \left| 0 \right>_e \right. \]
\[ + 2 i \left( \frac{e^{-i \phi} + e^{i \phi}}{2} \right) \left| 10 \right>_m \left| 11 \right>_e \left. \right] \]

\[ = ye^{i \phi/2} \left| 11 \right>_e \left[ \left( -e^{-i \phi/2} \left| 11 \right>_m \left| 0 \right>_e \right. \right. \]
\[ + 2 i \left( e^{-i \phi/2} + e^{i \phi/2} \right) \left| 10 \right>_m \left| 11 \right>_e \left. \right] \]

So what is \( P_{\text{coin}} (1e, 1k) \)? Detact 1 at \( D_s \) and 1 at \( D_i \)

\[ P_{\text{coin}} (1e, 1k) = \frac{1}{N} \left| \left\langle \psi \left| a_1^\dagger a_1^\dagger \left| \psi \right. \right. \right. \right| \]
\[ = \frac{\lambda^2}{\sqrt{2}} \left( 1 + \cos \Phi \right) \]
So what is 
\[ P(\text{le}) \text{ independent/ignorance/trace } D_i? \]

\[
P_s(\text{le}) = \left< \psi | a^\dagger \text{e} | \psi \right> = \sqrt{\frac{\pi^2}{2}} \left( 1 + \cos \phi \right)
\]

same! \( D_i \) does not even need to be there!

Okay we move Beam Block B into place.

Now \( |11\text{e}10\text{e} \neq |10\text{e}11\text{e} \) modes are distinguishable! They no longer factorize.

\[
|\psi\rangle_s = \frac{\sqrt{2}}{2} \left[ |11\text{e}10\text{e}\rangle + e^{i\phi} |0\text{e}11\text{e}\rangle + e^{i\phi} |10\text{e}11\text{e}\rangle + e^{i\phi} |11\text{e}10\text{e}\rangle \right]
\]

Note \( |11\text{e}10\text{e}\rangle \perp \text{ to } |10\text{e}11\text{e}\rangle \)

\[
P_{\text{com}}(\text{le}1\text{le}) = \frac{1}{2} \left< \psi | a^\dagger \text{e} a^\dagger \text{le} a^\dagger \text{le} | \psi \right>
\]

= \[ \frac{\pi^2}{4} \quad \text{Interference vanishes} \]

\( D_i \) can now tell which path \( 1\text{le} \) took.

Even better.
\[ P_s(1_e) = \frac{1}{2} \langle \psi | a^+ a_1 | \psi \rangle \frac{1}{\sqrt{2}} \]

\[ = \frac{\gamma}{4} + \frac{\gamma}{4} = \frac{\gamma}{2} \]

**Still no interference**

D\textsubscript{i} does not have to even be there!

The fact that which-path information is available in environment destroys interference even if nobody looks!

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The really crazy paper

Replace D\textsubscript{i} with a black hole

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Event horizon (in some theories) acts as quantum eraser and erases which-path.

Interference comes back! E.W.M

Interferometer can be used as a black hole detector! (Hockney & Yurtsever)