The goal is to compare QM× to LHVΘ

Polarization Rotators

**ALICE**

\[ |\Theta_1^A\rangle = \cos \Theta |H\rangle_1^A + \sin \Theta |V\rangle_1^A \]

\[ |\Theta_2^A\rangle = -i \sin \Theta |H\rangle_1^A + \cos \Theta |V\rangle_1^A \]

**BOB**

\[ |\phi_1^B\rangle = \cos \phi |H\rangle_2^B + \sin \phi |V\rangle_2^B \]

\[ |\phi_2^B\rangle = -i \sin \phi |H\rangle_2^B + \cos \phi |V\rangle_2^B \]

Since \( \phi = \text{BOB} \) and \( \Theta = \text{ALICE} \), we drop \( A, B \) and \( i, j \).
Note the inverse: $|H, \psi\rangle \rightarrow |\Theta, \bar{\Theta}\rangle$

**Alice**

$|H\psi\rangle^A = \cos \Theta \; |\Theta\rangle - i \sin \Theta \; |\bar{\Theta}\rangle$

$|V\psi\rangle^A = \sin \Theta \; |\Theta\rangle + \cos \Theta \; |\bar{\Theta}\rangle$

**Bob**

$|H\psi\rangle^B = \cos \Phi \; |\Phi\rangle - i \sin \Phi \; |\bar{\Phi}\rangle$

$|V\psi\rangle^B = \sin \Phi \; |\Phi\rangle + \cos \Phi \; |\bar{\Phi}\rangle$

**Hence we can write**

$|\psi\rangle = \frac{|H\psi\rangle^B |V\psi\rangle^B - |V\psi\rangle^H |H\psi\rangle^B}{\sqrt{2}}$

**Then**

$= \frac{1}{\sqrt{2}} \left[ -2i \; [ \Theta \Phi ] \; |\Theta\rangle |\Phi\rangle \\
+ 2 \; [ \Theta - \Phi ] \; |\Theta\rangle |\bar{\Phi}\rangle \\
- 2 \; [ \Theta - \Phi ] \; |\bar{\Theta}\rangle |\Phi\rangle \\
- 2i \; [ \Theta - \Phi ] \; |\bar{\Theta}\rangle |\bar{\Phi}\rangle \right]$

**Note** $\Theta = \Phi$

$\Rightarrow |\psi\rangle = \frac{10 \; |\bar{\Phi}\rangle - 1 \; |\bar{\Phi}\rangle |\phi\rangle}{\sqrt{2}}$

That is when $A$ and $B$ measure in *same basis* find 100% anti-correlation $|\psi\rangle$ invariant under basis change!
Let Alice construct
\[
\hat{\alpha}_x (\theta) = |0\rangle\langle 0| + |\bar{1}\rangle\langle 1|
\]
\[
\hat{\alpha}_y (\theta) = -i [ |0\rangle\langle 0| - |\bar{1}\rangle\langle 1|]
\]
\[
\hat{\alpha}_z (\theta) = |0\rangle\langle 0| - |\bar{1}\rangle\langle 1|
\]

Similar for Bob
\[
[ \hat{\alpha}_x, \hat{\alpha}_y ] = 2i \hat{\alpha}_z
\]

are pauli spinors so \( H, V \equiv \uparrow \downarrow \) spin \( \frac{\sqrt{2}}{2} \)

The point is A and B can decide which of the two measurements to make long after \( T = 2L/c \). Let us suppose measure \( \hat{\alpha}_z (\theta) = +1 \)

She concludes photon was in \( |0\rangle_A \)
Hence if Bob measures also in same basis \( \langle \hat{\alpha}_z (\theta) \rangle = -1 \) \( \langle \hat{\alpha}_z (\bar{1}) \rangle = 1 \) however on \( |\bar{1}\rangle_B \)
we can not conclude in QM that Bob's state was definitely in \( |\bar{1}\rangle_B \)

before meas. He could have chosen
\[
\langle \hat{\alpha}_x (\theta) \rangle = +1 \quad \text{in which case he would conclude} \quad \frac{|0\rangle_B + |\bar{1}\rangle_B}{\sqrt{2}}
\]

In QM Bob's photon has no state

\( \text{unreal} \)
Alice chooses $\Theta = 0$ and Bob chooses $\Theta = 0$. The sequences are:

- Alice: $H$, $V$, $H$, $V$
- Bob: $V$, $H$, $V$, $H$

Anticorrelation always holds, so how does Bob’s photon “know” which basis Alice used? Non-local!

Firstly, consider:

Let $\hat{A}(\Theta) = \left\{ \begin{array}{c|c} 1 & 10 \\hline 1 & 1\Theta \end{array} \right\} = \frac{\hat{\sigma}_A^X(\Theta)}{2}$

$\hat{B}(\Theta) = \left\{ \begin{array}{c|c} 1 & 1\phi \\hline 1 & 1\phi \end{array} \right\} = \frac{\hat{\sigma}_B^Y(\phi)}{2}$

We define correlation:

$$C[\Theta, \Phi] \equiv \text{Avg} \{ A(\Theta) B(\Phi) \}$$

In general:

$$C[\Theta, \Phi] \equiv \left< \psi \left| \hat{\sigma}_A^X(\Theta) \hat{\sigma}_B^Y(\Phi) \right| \psi \right>$$

Trig:

$$= -\cos[2(\Theta - \Phi)]$$

Recall $\Theta = \Phi$ so:

$$C[\Theta, \Phi] = -1$$

100% perfect anti-correlation
Now assume \( A; B \) are classical random variables
\( A(\Theta, \lambda) = \pm 1 \)
\( B(\Phi, \lambda) = \pm 1 \)

\[ \text{hidden variable} \]

**REALISM POSTULATE**

\( \forall \lambda \ A(\Theta, \lambda) \) and \( B(\Phi, \lambda) \) have definite values \( \pm 1 \) at all times for each \( \lambda \)

**LOCALITY POSTULATE**

\( A(\Theta, \lambda) \) does not depend on \( \Phi \)
\( B(\Phi, \lambda) \) does not depend on \( \Theta \)

\( \exists p(\lambda) \text{ s.t. } \int p(\lambda) \, d\lambda = 1 \)

\[ \text{PROB DIST} \]

\[ \text{LET } x, y, \bar{x}, \bar{y} \in \mathbb{R} \pm 1 \]

\[ S = xy + x\bar{y} + \bar{x}y - x\bar{y} \]

\[ C(\Theta, \Phi) = \int_{\text{LHV0}} p(\lambda) A(\Theta, \lambda) B(\Phi, \lambda) \, p(\lambda) \, d\lambda \]

\[ \text{CLASSICAL AVG} \]
Let \( x \times y \times y' \in \mathbb{E} \pm 13 \)

\[
S = xy + xy' - x'y' = \pm 2
\]

If we take

\[
x = A(\theta, \lambda) \quad x' = (\theta', \lambda)
\]

Two different settings A POL

\[
y = B[\phi, \epsilon] \quad y' = (\phi', \epsilon)
\]

Bob's POL

Idea is in one run \( \Theta, \phi \)

Another run \( \Theta', \phi' \)

\[
\Rightarrow -2 \leq S_{\text{ap}(a)} S(\theta, \phi, \phi, \lambda) \leq 2
\]

Since \( 0 \leq r(a) \leq 1 \)

\[
\Rightarrow -2 \leq C_{\text{unr}}(\theta, \phi) + C_{\text{r}}(\theta', \phi') + C_{\text{h}}(\theta, \phi') - C_{\text{h}}(\theta, \phi) \leq 2
\]

\[\forall \theta, \phi, \phi' \]

Now take \( \Theta = 0, \Theta' = \pi/4, \phi = \pi/2, \phi' = -\pi/2 \)

Non orthogonal

\[
S_{\text{Qmx}} = \frac{C(\theta, \phi) + C(\theta', \phi') + C(\phi, \phi') - C(\phi', \phi)}{Q}
\]

\[\Rightarrow -2 \sqrt{2} < -2\]
Detector loop hole

\[ \frac{I_A}{D} \rightarrow 3I_A \]

\[ \frac{I_B}{D} \rightarrow 3I_B \]

\[ C(\theta, \phi) = \frac{1}{2^2} \sec \left[ (\theta^2 - \phi) \right] \]

Fair sampling

\[ \sin x = -\frac{1}{2} \sqrt{2} < -2 \]

\[ 2^{-4} \cdot 2 = 4 \]

\[ 2^4 = \frac{1}{2} \]

\[ \varepsilon = \frac{1}{2^4} = 0.0625 \]

Fair sampling

If \(< 84\% \) of photons detected then these are far sample of missing photons!

LHCO charmless in detectors could shuffle data!
Light cone loop hole

A and B must choose \( \Theta, \Phi \)
outside of each other's light cones.

Super Fast Gremlin in \( s, a, b \)
conspire in UVW.
DET. LOOPTHOLE CLOSED IN L0N TRG03

L.C. LOOPTHOLE CLOSED IN PHOTON EXP

BOTH NOT CLOSED YET