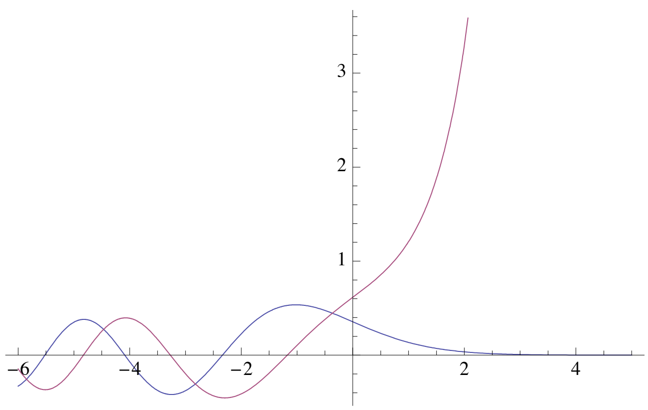
```
(*As in the notes,the scaled time-independent Schrödinger equation
  for the particle bouncing off the floor in a gravitational field is:*)

DSolve[ψ''[z] + (ϵ - γ * z) ψ[z] == 0, ψ[z], z]
```

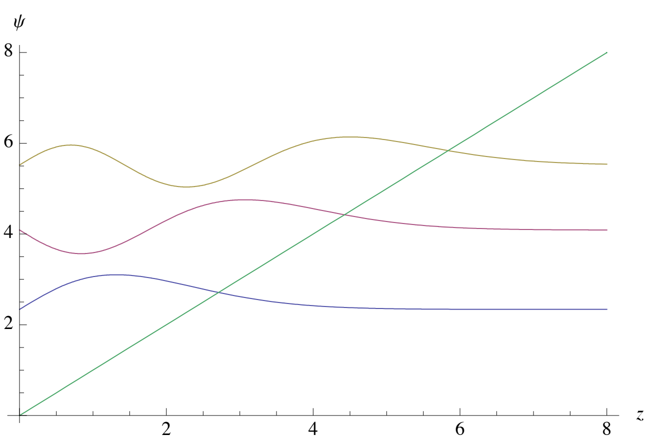
$$\left\{\left\{\psi[z] \to \text{AiryAi}\left[\frac{z\,\gamma - \epsilon}{\gamma^{2/3}}\right] C[1] + \text{AiryBi}\left[\frac{z\,\gamma - \epsilon}{\gamma^{2/3}}\right] C[2]\right\}\right\}$$

```
(*Thus by choosing γ=1,
we can rescale the mass so in the scaled units ϵ is the scaled energy and z is the scaled
  height of the atom off the floor. The general solution is then ψ(z)=a*Ai(z-ϵ)+b*Bi(z-ϵ),
where ϵ is the scaled energy, z is the scaled height of the atom off the floor,
Ai and Bi are the Airy function solutions to Bessel's equation,
and a and b are constants of integration. Let's plot the Airy Functions:*)

Plot[{AiryAi[x], AiryBi[x]}, {x, -6, 5}]
```




```
(*The Ai in blue converges for large z like Exp[-z] but the Bi in red diverges like
  Exp[+z]. Hence to get a physical solution we must impose the boundary condition b=
 0 to eliminate the Bi solution and hence ψϵ(z)=a*Ai(z-ϵ). Our second boundary condition
   is to demand that the wavefunction vanish at the floor, where the potential is infinite,
that is,  ψϵ(0)=a*Ai(-ϵ)=0 This quantizes the energies to be the roots of the Ai
   function. Looking at the above blue plot we can guess the first three roots
   and use the root finder to get the exact values for the first three roots.

FindRoot[AiryAi[x], {x, -2}]

{x → -2.33811}

FindRoot[AiryAi[x], {x, -4}]

{x → -4.08795}

FindRoot[AiryAi[x], {x, -6}]

{x → -5.52056}

(*Using these results, we can write down the first three quantized energy eigenvalues.*)

ϵ1 = 2.338107410459767

2.33811

ϵ2 = 4.08794944413097

4.08795
```

```
ϵ3 = 5.520559828095552

5.52056

(*These are all positive since we take V(z)=
 mgz to be zero at the floor where z=0. Corresponding to these three energies,
there will be three eigenfunctions ψ₁, ψ₂, and ψ₃,
with corresponding normalization constants, a₁, a₂,
and a₃. To find the normalization constants we integrate |ψ|² from z=
 0 to z=∞ and then divide by the square root of the result.*)

a1 = 1 / Sqrt[NIntegrate[AiryAi[z - ϵ1]^2, {z, 0, Infinity}]]

1.4261

a2 = 1 / Sqrt[NIntegrate[AiryAi[z - ϵ2]^2, {z, 0, Infinity}]]

1.24516

a3 = 1 / Sqrt[NIntegrate[AiryAi[z - ϵ3]^2, {z, 0, Infinity}]]

1.1558

(*We now plot the three normalized eignefunctions and the potential,
 which in these scaled units, has the form V(z)=z. As is usually done
   we displace the wavefunctions vertically by an amount of ϵi for clarity.)

ψ1[z_] := ϵ1 + a1 * AiryAi[z - ϵ1]

ψ2[z_] := ϵ2 + a2 * AiryAi[z - ϵ2]

ψ3[z_] := ϵ3 + a3 * AiryAi[z - ϵ3]

Plot[{ψ1[z], ψ2[z], ψ3[z], z}, {z, 0, 8}, AxesLabel → {z, ψ}]
```




```
(*The blue, red, and yellow curves are ψ1[z], ψ2[z],
and ψ3[z]. The green curve is V(z)=z. Classically a bouncing ball is confined between the y-
  azis and the green line. The green line is the turn-around point where the ball
   reaches its maximum height. The quantum particle tunnels into the forbidden non-
  classical regime to the right of the green line where it becomes a decaying
   exponential! Hence if you drop the atom from 2.0 microns there is a non-
  zero probability of finding it at 2.1 microns later on. This is impoosible
   classically. Between the y-axis and the green potential line,
the wavefunction oscillates like a standing wave. Also classically
 any energy is allowed where quantum mechanically
 only energies that are roots of Ai are allowed.*)
```